MA 323A **2/14/01**

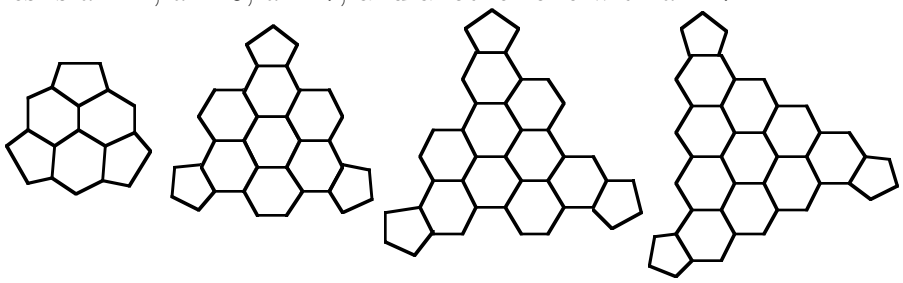
Combinatorial Geometry
Doin' the Problem Set Drei

(1) Prove rigorously, that the area of a spherical triangle with interior angles α_1, α_2, α_3 on a sphere with radius r is

$$\text{Area of triangle} = (\alpha_1 + \alpha_2 + \alpha_3 - \pi)r^2$$

(2) Use the result in problem (1) to come up with a formula for the area of a **spherical n-gon**. (Like, a spherical quadrilateral, or pentagon, etc.)

(3) Below are shown four "tiles" that can make spherical Buckyballs (where we put one tile on each face of the icosahedron). The distance d between neighboring pentagons in these tiles is $d = 2$, $d = 5$, $d = 7$, and another one with $d = 7$.



How many different tiles are possible for $d = 2, 3, 4, 5, 6, 7, ...$? Come up with a way to describe what these tiles can look like.

(4) Consider the polyhedral family of **Luckyballs**. A Luckyball is a polyhedron where

- each vertex has degree 3 and
- each face is either a square or a hexagon.

How many square faces must every Luckyball have? Prove it.

(5) Not for write-up: Look at my web page

`http://web.merrimack.edu/~thull/OrigamiMath.html`

and make the "Five Intersecting Tetrahedra" model. Putting it together is a real puzzle – you may want to print out the web pages and use the pictures as a guide.

(6) What is the "optimal" strutt width we would need to make the five intersecting tetrahedra using tetrahedral frames? See the following picture for inspiration.