MA 323 **2/7/02**

Combinatorial Geometry
Trippy Problem Set Number Two

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(1) The Four Color Theorem is very hard to prove, but the Five Color Theorem isn't so bad. Try to prove that you can color the vertices of any planar graph using five colors. (Feel free to ask for hints! Here's one: start with the fact that any planar graph must have a vertex of dedgree 5 or less.)

(2) Prove that a graph is 2-vertex colorable if and only if it contains no odd cycles. (This is sometimes called **The Two Color Theorem**.

(3) Classify the spherical Buckyballs. That is, how can you describe the spherical Buckyball polyhedra family? How many vertices, edges, and faces does each member of this family have? Can you smell any equations? (Note: this is a hard problem - there are many more spherical Buckyballs out there than I describe on my web page.)

(4) Try making a large Buckyball. Tom suggests starting with the soccer ball, BUT you gotta have a plan to 3-color it first! Include a description of your coloring strategy in your write-up. (Hint: you might try looking for a Hamilton cycle.)

(5) Prove the conjecture that we developed in class about the Vertex-Face Chart: that all polyhedra must live in the "wedge" region.

(6) Can you use Sonobé units to make a **torus** (i.e. a doghnut)??? What's the fewest number of colors that you need to properly color it?

(7) Is the below planar graph a polyhedron? Why or why not?